An Electroweak Model With Massive Neutrinos

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PACS Numbers: 14.60.Pq, 13.15+g, 12.15-y, 11.15-q, 12.10-g
Keywords: neutrino mass, electroweak interactions, gauge symmetry

[Abstract] In this paper, an electroweak model with massive neutrinos is proposed. Because of the mixing of neutrinos, the conversion of one lepton type to another is possible, but many of those kind processes are of extremely small possibilities. The total lepton-number conservation still holds in this model, so neutrinoless double beta decay is prohibited. Though right-hand neutrinos are introduced into the model, their coupling with matter fields is extremely small and therefore they are much harder to be detected in experiments.

Recently, strong evidence that neutrinos do have nonzero masses has been found[1-4]. But in the Standard Model, neutrinos are massless. In order to explain neutrino oscillation, some scenarios were put forward[5,6]. But in most of the models which contain massive neutrinos, the weak interactions of leptons are quite different from those of quarks. If the Nature has beautiful symmetry, quarks and leptons should have quite similar properties in weak interactions, and the fundamental theories on the electroweak interactions of quarks and leptons should have the same structure. On the other hand, it is known that, in the Standard Model, all quarks have right-hand components but only charged leptons have right-hand components. What cause this asymmetry? In another words, why neutrinos have no right-hand components while all quarks have right-hand components? A fundamental theory should not have so much arbitrariness and the fundamental rules for weak interactions of quarks and leptons should be the same. On the basis of this consideration, we will propose a new electroweak model in this paper. This model inherits most of the dynamical properties of the Standard Model but contains massive Dirac neutrinos and is consistent with experiments on weak interactions. More important, according to this model, the fundamental rules on the electroweak interactions of quarks and leptons are the same, and the fundamental theories on their electroweak interactions have the same structure in form[7]. As an example, we only discuss leptons in this paper.

In order to do this in a more fundamental manner, let's discuss the symmetry of the Nature first. It is known that the underlying symmetries of the Nature take a fundamental role in constructing the Standard Model. Two underlying symmetries are used in the Standard Model, they are $SU(2)_L$ and $U(1)_Y$[8-10]. But there is another underlying symmetry of the Nature which is not fully used in constructing the Standard Model. This symmetry corresponds to one of the most strict conservation laws of the Nature which had been strictly tested by experiments: the total lepton number conservation and total baryon number conservation. Because leptons and quarks are usually called matter fields, we call this conservation matter number conservation. This is a $U(1)$ symmetry and is denoted as $U(1)_M$ at present. So, the symmetry of this model is supposed to be $SU(2)_L \times U(1)_Y \times U(1)_M$. We supposed that all leptons and quarks carry matter number 1, all anti-leptons and anti-quarks carry matter number -1, and vacuum carries no matter number. Correspondingly, we need two kinds of gauge fields $F_{1\mu}$ and $F_{2\mu}$ which correspond to $SU(2)_L$ symmetry, two kinds of gauge fields $B_{1\mu}$ and $B_{2\mu}$ which correspond to $U(1)_Y$ and $U(1)_M$ symmetry respectively, and a third $U(1)$ gauge field $B_{3\mu}$ which connects thess two $U(1)$ symmetries. The coupling between leptons and gauge field $B_{1\mu}$ is determined by supercharge Y and the coupling between leptons and gauge field $B_{2\mu}$ is determined by matter number M. Under $U(1)_Y$ gauge transformations, two $U(1)$ gauge fields $B_{1\mu}$ and $B_{3\mu}$ will have to transform accordingly, while under $U(1)_M$ gauge transformations, two $U(1)$ gauge fields $B_{2\mu}$ and $B_{3\mu}$ will have to transform. So, there are five different kinds of gauge covariant derivatives:

$$
\begin{array}{l}
D_{1\mu} = \partial_\mu - igF_{1\mu}, \\
D_{2\mu} = \partial_\mu + ig\mathrm{tg}\alpha F_{2\mu}, \\
D_{3\mu} = \partial_\mu - ig_1 B_{1\mu}\frac{Y}{2}, \\
D_{4\mu} = \partial_\mu - ig_2 B_{1\mu}\frac{M}{2}, \\
D_{5\mu} = \partial_\mu + ig'\mathrm{tg}\alpha B_{2\mu},
\end{array} \tag{1}
$$

where α is a dimensionless parameter and

$$
g' = \sqrt{g_1^2 + g_2^2}. \tag{2}
$$

Let $e^{(i)}$ represent e, μ or τ, and $\nu^{(i)}$ represent the corresponding neutrinos ν_e, ν_μ or ν_τ. That is

$$
\begin{array}{l}
e^{(1)} = e, \quad e^{(2)} = \mu, \quad e^{(3)} = \tau, \\
nu^{(1)} = \nu_e, \quad ,\nu^{(2)} = \nu_\mu, \quad \nu^{(3)} = \nu_\tau.
\end{array} \tag{3}
$$

Leptons $e^{(i)}$ and $\nu^{(i)}$ form left-hand doublets $\psi_L^{(i)}$ and right-hand singlets $e_R^{(i)}$ and $\nu_R^{(i)}$. The definitions of ψ_L,

e_R and ν_R are the same as those in the Standard Model. The mixing of neutrinos is accomplished through:

$$\begin{pmatrix} \nu_\theta^{(1)} \\ \nu_\theta^{(2)} \\ \nu_\theta^{(3)} \end{pmatrix} = K \begin{pmatrix} \nu^{(1)} \\ \nu^{(2)} \\ \nu^{(3)} \end{pmatrix}, \tag{4}$$

where K is the mixing matrix for neutrinos.

The Lagrangian of the model is

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_g + \mathcal{L}_{v-l} \tag{5}$$

$$\begin{array}{l} \mathcal{L}_l = -\sum_{j=1}^3 \overline{\psi}_L^{(j)} \gamma^\mu (\partial_\mu - igF_{1\mu} \\ +\frac{i}{2}(g_1 B_{1\mu} - g_2 B_{2\mu}))\psi_L^{(j)} \\ -\sum_{j=1}^3 \overline{e}_R^{(j)} \gamma^\mu (\partial_\mu + i(g_1 B_{1\mu} - \frac{g_2}{2} B_{2\mu}))e_R^{(j)} \\ -\sum_{j=1}^3 \overline{\nu}_{\theta R}^{(j)} \gamma^\mu (\partial_\mu - \frac{i}{2} g_2 B_{2\mu})\nu_{\theta R}^{(j)} \end{array} \tag{6}$$

$$\begin{array}{l} \mathcal{L}_g = -\frac{1}{4}F_1^{i\mu\nu}F_{1\mu\nu}^i - \frac{1}{4}F_2^{i\mu\nu}F_{2\mu\nu}^i - \frac{1}{4}B_1^{\mu\nu}B_{1\mu\nu} \\ -\frac{1}{4}B_2^{\mu\nu}B_{2\mu\nu} - \frac{1}{4}B_3^{\mu\nu}B_{3\mu\nu} \\ -v^\dagger[\cos\theta_W(\cos\alpha F_1^\mu + \sin\alpha F_2^\mu) \\ -\sin\theta_W(\cos\alpha(\cos\beta B_{1\mu} - \sin\beta B_{2\mu}) + \sin\alpha B_3^\mu)] \\ \cdot[\cos\theta_W(\cos\alpha F_{1\mu} + \sin\alpha F_{2\mu}) \\ -\sin\theta_W(\cos\alpha(\cos\beta B_{1\mu} - \sin\beta B_{2\mu}) + \sin\alpha B_{3\mu})]v \end{array} \tag{7}$$

$$\begin{array}{l} \mathcal{L}_{v-l} = -\sum_{j=1}^3 (f^{(j)}\overline{e}_R v^\dagger \psi_L^{(j)} + f^{(j)*}\overline{\psi}_L^{(j)} v e_R) \\ -\sum_{j,k=1}^3 (f^{(jk)}\overline{\psi}_L^{(j)} \overline{v}\nu_{\theta R}^k + f^{(jk)*}\overline{\nu}_{\theta R}^k \overline{v}^\dagger \psi_L^{(j)}), \end{array} \tag{8}$$

where

$$\begin{array}{l} F_{1\mu\nu}^i = \partial_\mu F_{1\nu}^i - \partial_\nu F_{1\mu}^i + g\epsilon_{ijk}F_{1\mu}^j F_{1\nu}^k \\ F_{2\mu\nu}^i = \partial_\mu F_{2\nu}^i - \partial_\nu F_{2\mu}^i - g\mathrm{tg}\alpha\epsilon_{ijk}F_{2\mu}^j F_{2\nu}^k \\ B_{m\mu\nu} = \partial_\mu B_{m\nu} - \partial_\nu B_{m\mu} \quad (m = 1, 2, 3), \end{array} \tag{9}$$

v is a two-dimensional vacuum potential, $\overline{v}$ is given by

$$\overline{v} = i\sigma_2 v^* = \begin{pmatrix} v_2^\dagger \\ -v_1^\dagger \end{pmatrix}, \tag{10}$$

and β is a dimensionless parameter which is given by:

$$\sin\beta = g_2/g' \quad , \quad \cos\beta = g_1/g'. \tag{11}$$

It could be strictly proved that the above Lagrangian has strict local $SU(2)_L \times U(1)_Y \times U(1)_M$ gauge symmetry[7].

The symmetry breaking of the model is accomplished through the phase transition of the vacuum. We could suppose that the above Lagrangian describes a special state of matter which exists in the condition of extreme high temperature. This state is a special phase of vacuum and may exist at a moment of Big Bang. Alone with the decreasing of the temperature of the state, the phase transition of the vacuum occurs. After phase transition, the vacuum potential changes into:

$$v = \begin{pmatrix} 0 \\ \mu/\sqrt{2} \end{pmatrix}. \tag{12}$$

After symmetry breaking, only $U(1)_Q$ and $U(1)_M$ symmetries are preserved.

In eq(8), parameters $f^{(j)}$ and $F = (f^{(jk)})$ are selected as:

$$\begin{array}{l} F = \frac{\sqrt{2}}{\mu} K \begin{pmatrix} M_{\nu e} & & \\ & M_{\nu\mu} & \\ & & M_{\nu\tau} \end{pmatrix} K^\dagger \\ f^{(1)} = \frac{\sqrt{2}M_e}{\mu}, f^{(2)} = \frac{\sqrt{2}M_\mu}{\mu}, f^{(3)} = \frac{\sqrt{2}M_\tau}{\mu}. \end{array} \tag{13}$$

Gauge fields $F_{1\mu}, F_{2\mu}, B_{1\mu}, B_{2\mu}$ and $B_{3\mu}$ are not eigenvectors of mass matrix. In order to obtain the eigenvectors of mass matrix, three sets of field transformations are needed:

$$\begin{array}{l} B_{4\mu} = \cos\beta B_{1\mu} - \sin\beta B_{2\mu} \\ B_{5\mu} = \sin\beta B_{1\mu} + \cos\beta B_{2\mu}, \end{array} \tag{14}$$

$$\begin{array}{l} W_\mu = \cos\alpha F_{1\mu} + \sin\alpha F_{2\mu} \\ W_{2\mu} = -\sin\alpha F_{1\mu} + \cos\alpha F_{2\mu} \\ C_{1\mu} = \cos\alpha B_{4\mu} + \sin\alpha B_{3\mu} \\ C_{2\mu} = -\sin\alpha B_{4\mu} + \cos\alpha B_{3\mu}, \end{array} \tag{15}$$

and

$$\begin{array}{l} Z_\mu = \sin\theta_W C_{1\mu} - \cos\theta_W W_\mu^3 \\ A_\mu = \cos\theta_W C_{1\mu} + \sin\theta_W W_\mu^3 \\ Z_{2\mu} = \sin\theta_W C_{2\mu} - \cos\theta_W W_{2\mu}^3 \\ A_{2\mu} = \cos\theta_W C_{2\mu} + \sin\theta_W W_{2\mu}^3. \end{array} \tag{16}$$

Because these transformations are pure algebra operations, they do not affect the symmetry of the Lagrangian.

Finally, the Lagrangian density changed into the following form:

$$\begin{array}{l} \mathcal{L}_l + \mathcal{L}_{v-l} = -\sum_{j=1}^3 \overline{e}^{(j)}(\gamma^\mu\partial_\mu + M^{(j)})e^{(j)} \\ -\sum_{j=1}^3 \overline{\nu}^{(j)}(\gamma^\mu\partial_\mu + M_\nu^{(j)})\nu^{(j)} \\ +\frac{1}{2}\sqrt{g^2+g'^2}\sin 2\theta_W j_\mu^{em}(\cos\alpha A^\mu - \sin\alpha A_2^\mu) \\ -\sqrt{g^2+g'^2} j_\mu^z(\cos\alpha Z^\mu - \sin\alpha Z_2^\mu) \\ +\frac{\sqrt{2}}{2}ig(\sum_{j=1}^3 \overline{\nu}_{\theta L}^{(j)}\gamma^\mu e_L^{(j)})(\cos\alpha W_\mu^+ - \sin\alpha W_{2\mu}^+) \\ +\frac{\sqrt{2}}{2}ig(\sum_{j=1}^3 \overline{e}_L^{(j)}\gamma^\mu \nu_{\theta L}^{(j)})(\cos\alpha W_\mu^- - \sin\alpha W_{2\mu}^-) \\ -\frac{i}{2}g'\sin\beta\sum_{j=1}^3(\overline{e}_R^{(j)}\gamma^\mu e_R^{(j)} - \overline{\nu}_R^{(j)}\gamma^\mu\nu_R^{(j)}) \\ \cdot(-\sin\beta\cos\alpha(\sin\theta_W Z_\mu + \cos\theta_W A_\mu) \\ +\sin\beta\sin\alpha(\sin\theta_W Z_{2\mu} + \cos\theta_W A_{2\mu}) + \cos\beta B_{5\mu}) \end{array} \tag{17}$$

$$\begin{array}{l} \mathcal{L}_g = -\frac{1}{2}W_0^{+\mu\nu}W_{0\mu\nu}^- - \frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} - \frac{1}{4}A^{\mu\nu}A_{\mu\nu} \\ -\frac{1}{2}W_{20}^{+\mu\nu}W_{20\mu\nu}^- - \frac{1}{4}Z_2^{\mu\nu}Z_{2\mu\nu} - \frac{1}{4}A_2^{\mu\nu}A_{2\mu\nu} \\ -\frac{1}{4}B_5^{\mu\nu}B_{5\mu\nu} - \frac{\mu^2}{2}Z^\mu Z_\mu - \mu^2\cos^2\theta_W W^{+\mu}W_\mu^- + \mathcal{L}_{gI} \end{array} \tag{18}$$

where, $\mathcal{L}_{gI}$ only contains interaction terms of gauge fields. In the above relations, all strengths of gauge fields have the following form:

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{19}$$

where A represents $W,\ W_2,\ Z,\ Z_2,\ A,\ A_2$, or B_5. The definitions of some other arguments are:

$$\begin{array}{l} W^{\pm}_{m\mu} = \frac{1}{\sqrt{2}}(W^1_{m\mu} \mp iW^2_{m\mu}) \\ \qquad\quad (m = 1, 2,\ W_{1\mu} \equiv W_\mu), \\ j^{em}_\mu = -i(\overline{e}\gamma_\mu e + \overline{\mu}\gamma_\mu \mu + \overline{\tau}\gamma_\mu \tau), \\ j^Z_\mu = j^3_\mu - \sin^2\theta_W j^{em}_\mu. \end{array} \tag{20}$$

From the above results, it could be seen that, all leptons are massive, gauge bosons $W^{\pm}$ and Z are also massive, but gauge fields $A, A_2, Z_2, W_2^{\pm}$ and B_5 are massless. The mass relation between $W^{\pm}$ and Z is similar to that in the Standard Model. The current structures in this model, especially the charged current structures, are also similar to those in the standard model. Besides, it could be noticed that, there exist two different electromagnetic fields A_μ and $A_{2\mu}$, so there exist two different coupling constants of electromagnetic interactions.

$$e_1 = \frac{gg'}{\sqrt{g^2 + g'^2}}\cos\alpha\ ,\quad e_2 = \frac{gg'}{\sqrt{g^2 + g'^2}}\sin\alpha. \tag{21}$$

The real electromagnetic field in Nature is the mixture of these two electromagnetic fields A_μ and $A_{2\mu}$, and the effective coupling constant of the electromagnetic interactions should be

$$e^2 = e_1^2 + e_2^2\ ,\quad e = \frac{gg'}{\sqrt{g^2 + g'^2}}. \tag{22}$$

So the value of the parameter α doesn't affect the value of the effective coupling constant of electromagnetic interactions.

It is expected that the parameter α and β should be small, so the contributions come from massless gauge fields and right-hand neutrinos will be very small and the present model will not contradict with high energy experiments.

It is generally believed that neutrinos detected by experiments are left-hand neutrinos and right-hand antineutrinos. In this model, though right-hand neutrinos are introduced, but they only couple with neutral currents and the corresponding coupling constant is about $\sin^2\beta \times$ ordinary weak coupling constant. Because $\sin\beta$ is very small, any process contains right-hand neutrinos will be extremely weak. Therefore, neutrinos detected by experiments are mostly left-hand neutrinos, and right-hand neutrinos is very hard to be detected. The Nature still has no left-right symmetry and the P Parity is not conserved. Besides, right-hand neutrinos have no direct coupling with charged leptons, so almost all neutrinos generated from β decay are left-hand neutrinos. That make them more difficult to be found in experiments.

Because of neutrinos' mixing, the lepton-number is not separately conserved. But no total lepton-number violating term is introduced, so the total lepton-number is still conserved. In another word, one type of lepton can convert into another type of lepton through neutrino mixing, but the total lepton number is not changed in this process. So, any process which will change the total lepton number, such as double beta decay, is prohibited by this model. This selection rule originate from $U(1)_M$ symmetry.

Though lepton-number is not separately conserved, some lepton-number-violating processes, such as $\mu^- \to e^-\gamma$, $\tau^- \to \mu^-\gamma$ and $\tau^- \to e^-\gamma$, will be extremely weak. As an example, let's discuss $\mu^- \to e^-\gamma$. In this process, all neutrinos appear as inner lines of a Feynman diagram. Suppose that this μ^- changes into a left-hand neutrino and a W^- particle and both of them are inner lines of a Feynman diagram. The Feynman propagater for neutrino is $< 0|\overline{\nu}\nu|0 >=< 0|\overline{\nu}_L\nu_R|0 > + < 0|\overline{\nu}_R\nu_L|0 >$. If one end of propagater is $\overline{\nu}_L$, another end must be ν_R. But right-hand neutrinos have no direct interactions with electrons, nor with W^- particle, it has to interact with a neutral gauge boson to become $\overline{\nu}_R$ which is at one end of a new neutrino inner line. The other end of this neutrino inner line must be ν_L which interact with W^- to produce an electron. This diagram is at least at one-loop level. Besides, the right-hand neutrinos have very weak coupling with gauge bosons. Both factors will make the branch ratio of this process extremely small.

In this model, two things are accomplished simultaneously: the introduction of neutrino mass and the disappearing of Higgs particle through introducing a new but familiar charge — total lepton number and total baryon number, and another set of gauge fields. The structure of this model is completely determined by the symmetry of the model. This model is renormalizable[7]. If we did not add the restriction of minimal on the theory, we could also construct a lot of models which contain massive neutrinos and even the Higgs particle[7]. A minimum model that contains both massive neutrinos and Higgs particle is that[7]:

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_g + \mathcal{L}_{H-l} + \mathcal{L}_H \tag{23}$$

where $\mathcal{L}_H$ is the Lagrangian for Higgs fields which is the same as that in the Standard Model, $\mathcal{L}_l$ is given by eq(6), $\mathcal{L}_{H-l}$ is given by eq(8) but should replace vacuum potential with Higgs fields, and $\mathcal{L}_g$ is given by:

$$\mathcal{L}_g = -\frac{1}{4}F_1^{i\mu\nu}F_{1\mu\nu}^i - \frac{1}{4}B_1^{\mu\nu}B_{1\mu\nu} - \frac{1}{4}B_2^{\mu\nu}B_{2\mu\nu}. \qquad (24)$$

This model has some interesting properties but has more independent parameters than the previous one[7]. Besides, its structure is relatively more loosy.

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